Notes on Groups

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This is from my Euclidean/Non-Euclidean Geometry Textbook.

Group Axioms: You have a set of elements, say $p, q, r, ...$, and a binary operator. A binary operator combines two elements of a set to create another element of the set. The binary operator applied to p and q is written pq. An example of a binary operator is $+$. For a group, you also have the following conditions satisfied:

The associative law: $p(qr) = (pq)r$, for all p, q and r in the set.

The identity law: $ip = pi = p$ for all p and for some i in the set.

The inverse law: For any p in the set there exists a p' in the set such that $pp' = p'p = i$. We denote p' by p^{-1}.

If the group operation is commutative (that is, $pq = qp$ for all p, q in the group), then the group is said to be *Abelian*. In that case, the group operation is usually written as $p + q$ (I guess to denote this commutativity).